Superoscillatory Radar Imaging: Improving Radar Range Resolution Beyond Fundamental Bandwidth Limitations

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***Abstract*—In this work, we propose to improve the range resolution in a conventional radar system by employing a superoscillatory pulse as the radar pulse. A superoscillatory waveform is a waveform which contains, across a finite time interval, faster variations than its highest constituent frequency component. As such, radar imaging using a superoscillatory pulse allows one to detect an object with a range resolution improved beyond a fundamental bandwidth limitation. In this work, we experimentally compare the radar resolution performance of a 500 MHz superoscillatory pulse with that of a sinc pulse of the same bandwidth, and demonstrate that the superoscillatory pulse reduces distance uncertainty by 36%. We also suggest future directions of development to our proposed radar system.**

***Index Terms*—High-resolution imaging, radar detection, radar imaging, super-resolution.**

I. Introduction

IN a conventional radar scheme without detection filtering, the range resolution is related to the temporal width of the radar pulse by the simple relation

$$R_{3dB} = \frac{c_0 \Delta T_{3dB}}{2} \tag{1}$$

where c_0 is the speed of light in free space and R_{3dB} and ΔT_{3dB} are the 3 dB resolution and temporal width of the radar pulse, respectively. When a hard bandwidth limit is present, radar imaging with pulse having a uniform amplitude spectrum is perceived to achieve the best range resolution. The simplest pulse with a uniform amplitude spectrum across the bandwidth B is the sinc pulse. For this pulse, $\Delta T_{3dB} B = 0.443$, so the 3 dB resolution is related to the pulse bandwidth B as

$$R_{3dB} = \frac{c_0 \Delta T_{3dB}}{2} = \frac{0.443 c_0}{2B}. \tag{2}$$

While the chirped, flat-top pulse offers improved power performance, it provides a similar resolution as a simple sinc pulse after the matched filtering process [1]; hence it will not be separately considered in this work. In light of the inverse relation which generally occurs between the pulse bandwidth and the achievable range resolution, recent research effort is directed towards using UWB pulses for range detection in radar systems. Nonetheless, as bandwidth has become a precious commodity in modern communication systems, a way to circumvent the limit expressed in (2) is proved useful in high-range-resolution radar systems of various kinds.

In this work, we propose this alternative route to increased range resolution through the usage of superoscillatory waveforms. A superoscillatory waveform is one which contains a finite superoscillation interval, where oscillations occur faster than the waveform's highest constituent frequency component. Such oscillations allow the wave to temporarily behave as if it contains a higher effective bandwidth, which in turn allows the formation of sharper features than restricted by (2). Fig. 1(d), for example, shows an example of a superoscillatory waveform, whose 3 dB width is 35% reduced compared to that of a sinc pulse with the same bandwidth.

Though superoscillatory pulses have been theorized in the early 1990s [2], [3], they have not been experimentally demonstrated until very recently due to difficulties in managing sensitivity requirements. In a recent work [4], we overcame these difficulties to achieve a first demonstration of robust time domain superoscillatory waveforms, thus paving the way towards their application in radar systems. In the following, we first overview our method for superoscillatory pulse design. We then describe our experimental radar system, and compare the range resolution achieved using a superoscillatory pulse with that of a sinc pulse of the same bandwidth. Finally, we discuss the implications of this work and suggest directions for future improvement.

II. Superoscillatory Waveform Design

We adapted our method of superoscillatory waveform design from Schelkunoff's method of superdirective antenna design [4], [5]. We shall briefly overview our formulation, then employ it to design a superoscillatory radar pulse which will be used for radar imaging.

A. Overview of Formulation

Consider a periodic waveform synthesized by discrete sinusoidal components. Its frequency spectrum can be written in complex exponential form, as

$$\tilde{V}(\omega) = \sum_{n=0}^{N-1} a_n \delta(\omega - \omega_0 - n\Delta\omega) \tag{3}$$

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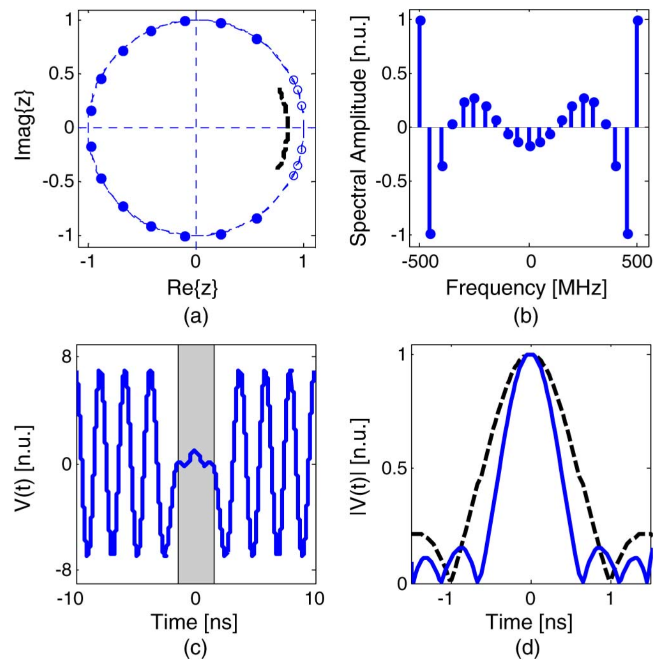

Fig. 1. Superoscillatory pulse design. (a) A plot showing the zeros for the superoscillatory pulse. Empty circles denote zeros within the superoscillatory region; filled circles denote zeros outside the superoscillatory region. The dotted curve line denotes the superoscillatory region. (b) The corresponding spectral amplitude. (c) One period of the superoscillatory pulse, with the shaded area denoting the superoscillatory region. (d) An amplitude plot of the superoscillatory region (solid), showing a 3 dB pulse width improvement of 35% over a sinc pulse of 500 MHz bandwidth (dashed).

where ω is the angular frequency, ω_0 represents the location of the lowest (most negative) frequency delta function, $\Delta\omega$ is the frequency spacing between adjacent tones, and a_n is the weight for the n'th delta function. The corresponding temporal waveform, obtained by taking the inverse Fourier transform of (3), is

$$V(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{V}(\omega)e^{j\omega t}d\omega = \frac{e^{j\omega_0 t}}{2\pi}\sum_{n=0}^{N-1} a_n z^n, z = e^{j\Delta\omega t}. \tag{4}$$

Equation (4) assumes a polynomial form, much like that for the array factor of an N-antenna array with feed current distribution a_n. Thus we can design its profile using methods of antenna array design. Following Schelkunoff's antenna design methodology, we factorize (4) to obtain

$$V(z = e^{j\Delta\omega t}) = \frac{a_{N-1}e^{j\omega_0 t}}{2\pi}\prod_{n=1}^{N-1}(z - z_n) \tag{5}$$

where z_n is the set of zeros for the function $V(z = e^{j\Delta\omega t})$, and where the voltage profile for a temporal Bloch period ($V(t)$ for $t_0 \le t \le t_0 + T$) corresponds to the value of $V(z)$ along one revolution around the unit circle $|z| = 1$. In this perspective, one can draw upon established works on discrete filter design [6] to place the $N-1$ zeros in the polynomial, and thereby design the temporal profile of the periodic waveform. This waveform synthesis method is perfectly analogous to Schelkunoff's method of antenna array synthesis.

A case of particular interest arises when one close-packs some or all available zeros into a design interval $T_d < T$. This way of placing zeros is analogous to the design of superdirective antennas, and will allow one to generate superoscillations within the time interval T_d.

B. *Waveform Design and Characterization*

Using this method, we design the superoscillatory pulse to be used in the proceeding experiment. We use 21 spectral lines evenly spread between ±500 MHz, which gives us freedom to locate 20 zeros on the z-plane. We locate six zeros within the superoscillatory design interval $T_d \in [-1.5\ \mathrm{ns}, 1.5\ \mathrm{ns}]$, using a Tschebyscheff polynomial expansion procedure. With this expansion we construct a waveform with sidelobes below 20% of the peak voltage. As a means of lowering the waveform sensitivity, we place the remaining 14 zeros outside the superoscillatory region to minimize the sideband amplitude. We suggest the interested reader to refer to [4] for a detailed discussion on our pulse design methodology.

Fig. 1(a) and 1(b) show the resulting zero locations and the corresponding spectral amplitude. Fig. 1(c) shows a period of the temporal waveform, with superoscillations occurring in the design region T_d. Fig. 1(d) compares the waveform to a sinc function of the same bandwidth across the design region T_d. The temporal waveform has a calculated 3 dB width of 0.57 ns, which is 35% narrower than the sinc pulse, for which the 3 dB width measures 0.88 ns. This pulse width can be further narrowed through tradeoffs with the duration of the design interval and the sideband amplitude. As we shall observe, this narrowed pulse width leads to a direct improvement in the range resolution in a radar imaging scheme.

III. Superoscillatory Radar System

We first generate our test pulse $V(t)$, as depicted in Fig. 1(c), with an arbitrary waveform generator bandlimited to 500 MHz. The arbitrary waveform generator combines the harmonics shown in Fig. 1(b) in the appropriate phases to form the desired $V(t)$. This pulse modulates a 4.2 GHz carrier, which is then amplified and transmitted through a horn antenna. The pulse is reflected off a metallic plate—representative of a point scatterer—placed a set distance away, and is recollected by the horn, demodulated and observed with an oscilloscope. Calibration with one scatter distance establishes a time of zero delay; thereafter, for a scatterer placed at an arbitrary distance away, the delay $\mathcal{T}$ attained by the superoscillatory peak determines the scatterer's distance through the simple relation

$$d = \frac{c_0\mathcal{T}}{2} \pm \frac{R_{3dB}}{2}. \tag{6}$$

Fig. 2 shows reflection traces at three image distances: 3.45 ± 0.02 m, 3.72 ± 0.02 m and 3.98 ± 0.02 m. These reflections translate into respective range measurements of 3.45 ± 0.05 m, 3.68 ± 0.05 m and 3.98 ± 0.06 m, which agree with results from physical measurements, and hence demonstrate the system's accuracy for single target detection. Notwithstanding slight distortions, the overall shapes of the superoscillatory pulses, including the superoscillatory peaks, are preserved.

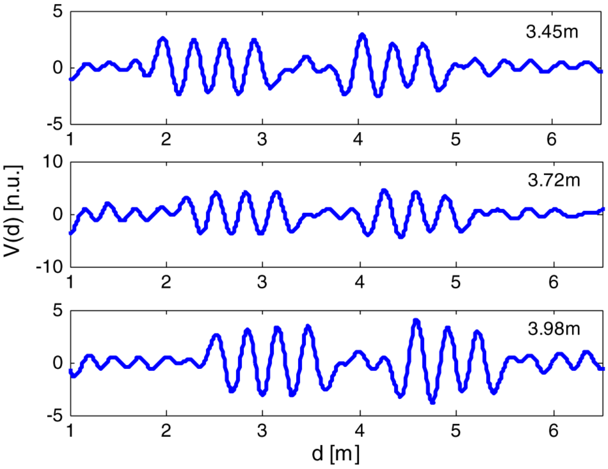

Fig. 2. Reflection traces for the superoscillatory pulse at distances 3.45 ± 0.02 m, 3.72 ± 0.02 m and 3.98 ± 0.02 m. The distances inferred from the peaks of the reflection signals are 3.45 ± 0.05 m, 3.68 ± 0.05 m and 3.98 ± 0.06 m respectively.

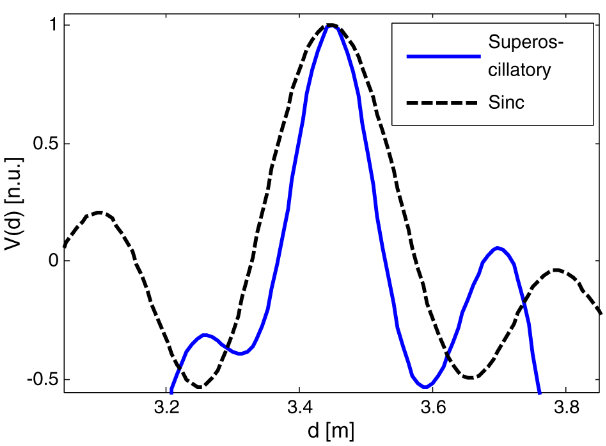

Fig. 3. Comparison on range resolution at $d = 3.45\ \mathrm{m}$. The 3 dB resolution for the superoscillatory pulse (solid) is 7.7 cm. This resolution is 36% improved from the measured 3 dB width of 12.0 cm for a 500 MHz sinc pulse reflected from the same distance (dashed).

Fig. 3 compares the superoscillatory pulse with a 500 MHz sinc pulse as they reflect from a scatterer placed at 3.45 m away from the horn. The superoscillation peak can be clearly observed, with a 3 dB resolution of 7.7 cm, which is 36% improved over the 500 MHz sinc radar pulse, which has a 3 dB resolution of 12.0 cm. This obtained percentage improvement agrees with our theoretical calculation in Section II, and thus shows that the reduced temporal width directly translates into an improvement in distance resolution. The observed reflections differ from the designed waveform in two minor manners. Firstly, since our demodulator rejects the DC signal component, our observed reflections each contain a DC offset as compared to the designed pulse. Additionally, an asymmetric skew appears to the sidelobe levels on either side of the main peak, likely as a result of dispersion within the radar system. While these artifacts can be rectified using conventional signal processing techniques, they do not degrade the pulse resolution as is evident in Fig. 3.

IV. Conclusion

In this work we have demonstrated for the first time that one can improve the range resolution of a radar system using a superoscillatory radar pulse. In our proof-of-principle experiment, we demonstrated a 36% improvement in range resolution by using a superoscillatory radar pulse in place of a sinc pulse of the same bandwidth. This improvement is obtained in real-time, without the aid of super-resolution post-processing algorithms. The resolution can be further increased through tradeoffs with sideband amplitudes and overall pulse duration. Admittedly, in the proposed scheme, resolution improvement is obtained at the expense of the system's signal-to-noise ratio, as theorized by previous work in superoscillations [7]. Nonetheless, the proposed scheme could be directly of use in radar systems where range resolution is to be maximized in the presence of hard bandwidth constraints. Further investigations which potentially improve the system include the application of superoscillation to match filtering processes, the comparison to, or incorporation with, other super-resolution post-processing techniques, imaging extended objects, and bandwidth extension through truncating and perhaps reshaping the superoscillatory sidebands.

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